Name of College – S.S. College J-Bag

Camlin Page 1
Date 11/07/2020

Deptt – Mathematics

Topic – Ellipse (Analytical Geometry of 2-Dimension)

Date – 11-07-2020

B.Sc I (Sub)

By – Dr. Shree Nath Sharma

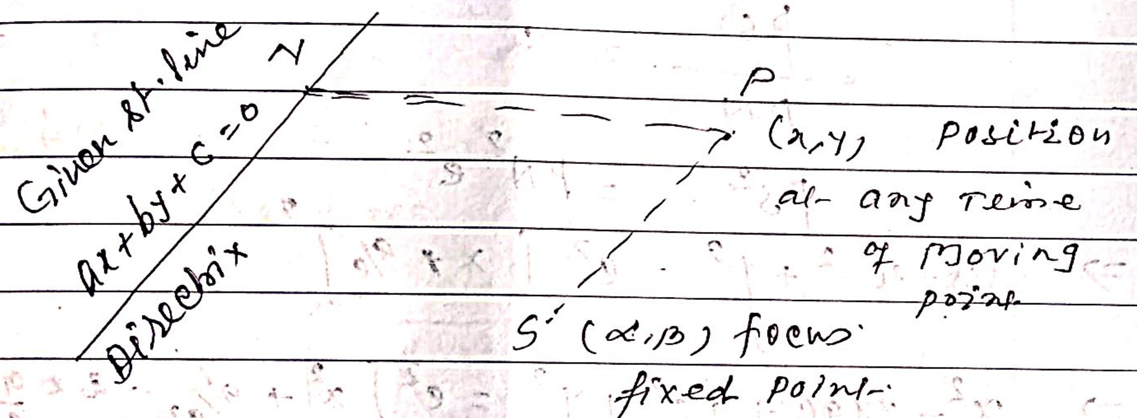
Ellipse →

It is the locus of the point which moves such that

$$\frac{\text{Distance of moving point from a fixed point}}{\text{Distance of moving point from a fixed st. line}} = e$$

where e = eccentricity < 1

General Equation :→



PN = Perpendicular distance of P from given st. line

$$= \frac{ax+by+c}{\sqrt{a^2+b^2}}$$

PS = Distance of P from fixed point $S(\alpha, \beta)$

$$= \sqrt{(x-\alpha)^2 + (y-\beta)^2}$$

Now since

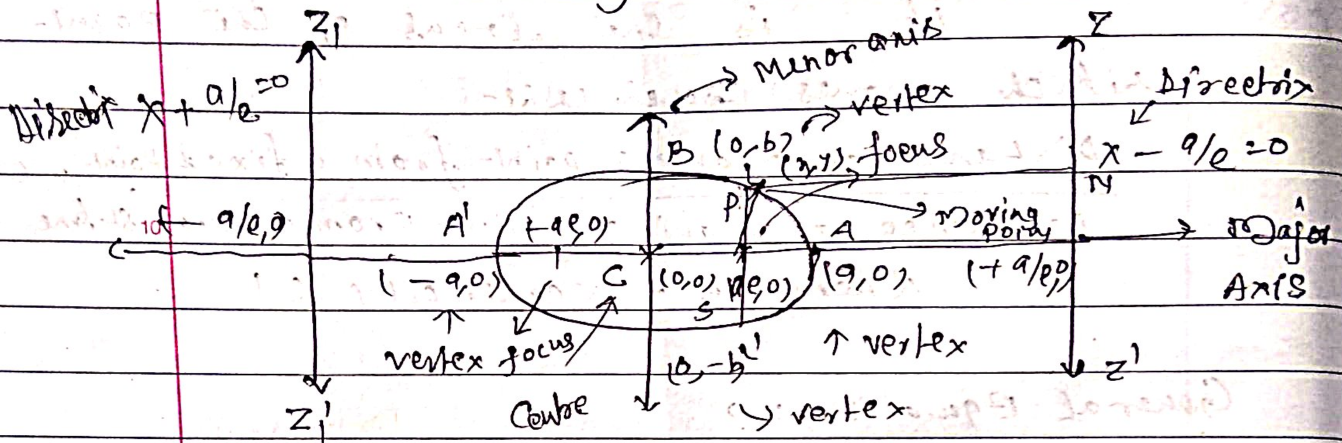
$$\frac{PS}{PN} = e \Rightarrow PS^2 = e^2 PN^2$$

$$\Rightarrow (x-\alpha)^2 + (y-\beta)^2 = e^2 \frac{(ax+by+c)^2}{a^2+b^2}$$

$$\Rightarrow \{(x-\alpha)^2+(y-\beta)^2\}\{a^2+b^2\} = e^2(ax+by+c)$$

This is the required General equation of ellipse.

Standard Equation of ~~Parabola~~ Ellipse.



$$\frac{PS}{PN} = e$$

$$PS^2 = PN^2 e^2$$

$$\Rightarrow (x-ae)^2 + y^2 = e^2\left[x + a/e\right]^2$$

$$\Rightarrow x^2 - 2xae + a^2e^2 + y^2 = e^2\left[x^2 + a^2/e^2 - 2x\frac{a}{e}\right]$$

$$\Rightarrow x^2 - 2xae + a^2e^2 + y^2 = x^2e^2 + a^2 - 2xae$$

$$\Rightarrow (x^2 - x^2e^2) + y^2 = a^2 - a^2e^2$$

$$\Rightarrow (1-e^2)x^2 + y^2 = (1-e^2)a^2$$

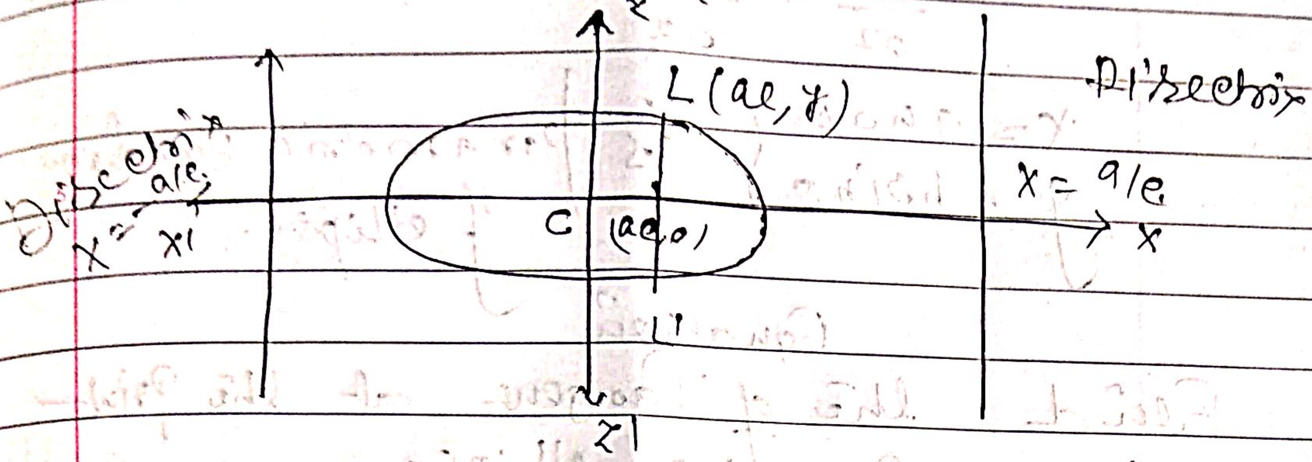
$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{a^2(1-e^2)} = 1$$

$$\Rightarrow x^2/a^2 + \frac{y^2}{b^2} = 1$$ where $b^2 = a^2(1-e^2)$

e = eccentricity < 1

This is the standard equation

Latus Rectum → A chord through focus parallel to directrix is called Latus Rectum



Since equal of ellipse.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{a^2(1-e^2)} = 1$$

Since L(ae, y) lies on ellipse therefore

$$\frac{\cancel{a^2}e^2}{\cancel{a^2}} + \frac{y^2}{a^2(1-e^2)} = 1$$

$$y^2 = a^2(1-e^2)^2$$

$$y = \pm a(1-e^2)$$

$$\Rightarrow y = \frac{a^2(1-e^2)}{a}$$

$$= \frac{b^2}{a}$$

∴ Length of latus rectum

$$= 2\cancel{\frac{b^2}{a}} \quad \frac{2b^2}{a}$$

Parametric Equation of ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$\left.\begin{array}{l} x = a\cos\theta \\ y = b\sin\theta \end{array}\right\}$ is Parametric Equation of ellipse.

Find the Equation of Tangent at the Point (x_1, y_1) on the ellipse.

Sol^n - $$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Equation of given ellipse is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad \text{—— (I)}$$

Since (x_1, y_1) be the Point of Contact

$$\Rightarrow \frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}=1 \quad \text{—— (II)}$$

Equation of Tangent at (x_1, y_1) is

$$\frac{y-y_1}{x-x_1} = \text{Slope of Tangent}$$

$$= \left(\frac{dy}{dx}\right)_{x_1, y_1} \quad \text{—— (III)}$$

Since $\frac{x^2}{a^2}+y^2/b^2=1$

Diff. with respect to x

$$\frac{2x}{a^2}+\frac{2y}{b^2}\frac{dy}{dx}=0$$

$$\therefore \frac{dy}{dx} = -\frac{b^2}{a^2}\cdot\frac{x}{y}$$

$$\therefore \left(\frac{dy}{dx}\right)_{x_1,y_1} = -\frac{b^2}{a^2}\cdot\frac{x_1}{y_1}$$

Thus From (iii)

Equation of tangent is

$$\frac{y-y_1}{x-x_1} = -\frac{b^2}{a^2}\frac{x_1}{y_1}$$

$$\Rightarrow (yy_1-y_1^2)a^2 = -b^2[xx_1-x_1^2]$$

$$\Rightarrow \frac{yy_1}{b^2}-\frac{y_1^2}{b^2} = -\frac{xx_1}{a^2}+\frac{x_1}{a^2}$$

$$\Rightarrow \frac{xx_1}{a^2}+\frac{yy_1}{b^2} = \frac{x_1^2}{a^2}+\frac{y_1^2}{b^2} = 1 \text{ (using (ii))}$$

$\Rightarrow$ Equation of Tangent at (x_1,y_1) is Given by

$$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$$

Equation of tangent at a the point φ

Since equation of Tangent at (x_1, y_1) is

$$\frac{x x_1}{a^2} + \frac{y y_1}{b^2} = 1$$

Put $x_1 = a\cos\varphi$
$y_1 = b\sin\varphi$

We get $\frac{x\, a\cos\varphi}{a^2} + \frac{y\, b\sin\varphi}{b^2} = 1$

$$\Rightarrow \frac{x\cos\varphi}{a} + \frac{y\sin\varphi}{b} = 1$$